ATMAM Unit 3 – Test 2 – 2017

Name: Solutions

Calculator Free Section (No notes or calculators. Formula sheet provided.)

Time allowed – 25 minutes

/23

Question 1 [2, 3, 3, 3, 4 marks]

a) Evaluate $\int_{-1}^{2} 2(x+1)^3\,dx$

$$= \cancel{2}\int_{-1}^{2} (x+1)^3\,dx$$

$$= \cancel{2}\times\left[\frac{(x+1)^4}{2\cancel{4}}\right]_{-1}^{2} \quad ✓$$

$$= \frac{81}{2} - 0$$

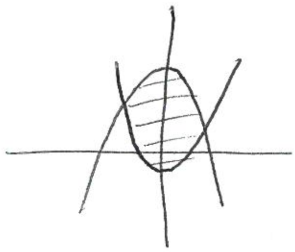
$$= \underline{40.5} \quad ✓$$

b) Find the area enclosed by the curve $y = x^2 - 1$ and $y = 7 - x^2$

$$x^2 - 1 = 7 - x^2$$

$$2x^2 = 8$$

$$x = \pm 2 \quad ✓ \text{ boundaries}$$



$$\int_{-2}^{2} 8 - 2x^2\,dx$$

$$= \left[8x - \frac{2x^3}{3}\right]_{-2}^{2} \quad ✓ \text{ anti-diff}$$

$$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$$

$$= 32 - \frac{32}{3}$$

$$= \underline{21\tfrac{1}{3}} \quad ✓ \text{ substitution}$$

c) Evaluate $f'(0)$ for $f(x) = xe^{3x}$

$$f'(x) = e^{3x} + x \cdot 3e^{3x}$$

✓ – uses product rule correctly

$$= e^{3x} + 3xe^{3x}$$

✓ – diff e^{3x} correctly

$$f'(0) = 1$$

✓ – substitution.

d) Find $\frac{d}{dx}\int_4^{e^x} e^{2\cancel{x}t}+3\cancel{x}t\ d\cancel{x}t$

$y = \int_4^u e^{2t}+3t\ dt \qquad u = e^x$ ✓ - using chain rule

$\frac{dy}{dx} = \left(e^{2u}+3u\right)e^x$ ✓

$= \left(e^{2e^x}+3e^x\right)e^x$ ✓

e) Show that $\int \frac{e^{5x}+3+e^x}{4e^{3x}}dx \ = \ \frac{e^{5x}-2-e^x}{8e^{3x}}+c$

$= \int \frac{e^{5x}}{4e^{3x}}dx + \int \frac{3}{4e^{3x}}dx + \int \frac{e^x}{4e^{3x}}dx$ ✓ - separates function

$= \frac{1}{4}\int e^{2x}dx + \frac{3}{4}\int e^{-3x}dx + \frac{1}{4}\int e^{-2x}dx$

$= \frac{e^{2x}}{8} + \frac{\cancel{3}e^{-3x}}{-\cancel{12}4} + \frac{e^{-2x}}{-8} + c$ ✓ - antidifferentiates

$= \frac{e^{2x}+2e^{-3x}+e^{-2x}}{8} + c$

$= \frac{e^{-3x}\left(e^{5x}+2+e^x\right)}{8} + c$

$= \frac{e^{5x}+2+e^x}{8} + c$ ✓ simplifies

Question 2 [4 marks]

Find the x-coordinate of all stationary points of the function $f(x) = \frac{(x-1)^2}{e^x}$

$f'(x) = \frac{e^x.2(x-1) - (x-1)^2e^x}{e^{2x}}$ ✓ - differentiates

$= \frac{(x-1)(2-x+1)}{e^x}$

$= \frac{(x-1)(3-x)}{e^x}$ ✓ - simplify

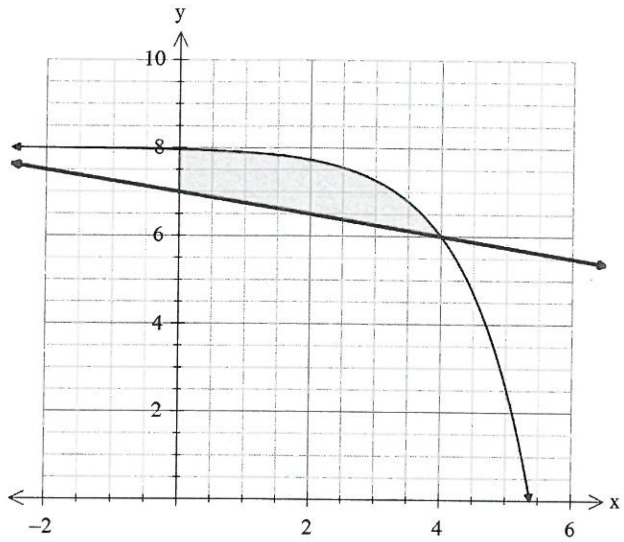
✓ $= 0$

Stat pts at $(x-1)=0$ and $(3-x)=0$

$\underline{x=1}$ and $\underline{x=3}$ ✓ solutions

Question 3 [4 marks]

The graphs of $y=8-2e^{x-4}$ and $y=-\frac{1}{4}x+5$ intersect at x=4 for x≥0 as shown. Determine the exact area between $y=8-2e^{x-4}$ and $y=-\frac{1}{4}x+5$ and the y-axis.



$$8-2e^{x-4}-\left(-\tfrac{1}{4}x+5\right)$$
$$=\frac{1}{4}x-2e^{x-4}+3 \quad ✓$$

$$\int_0^4 \frac{1}{4}x-2e^{x-4}+3\,dx$$

$$=\left[\frac{x^2}{8}-2e^{x-4}+3x\right]_0^4 \quad ✓ \quad ✓$$

$$=(2-2+12)-(0-2e^{-4}+0)$$

$$=12+\frac{2}{e^4} \quad ✓$$

ATMAM Unit 3 – Test 2 – 2017

CANNING VALE COLLEGE

Name:__________________________

Calculator Assumed Section (1 A4 page of notes allowed. Formula sheet provided.)

Time allowed – 30 minutes

$\frac{}{27}$

Question 4 [3 marks]

Water flows into a tank at a rate given by: $W'(t) = \frac{1}{75}(20t - t^2 + 600)$ where W'(t) is measured in L/hour and t is in hours. Initially, there are 200 L of water in the tank. How many litres of water are in the tank after 24 hours?

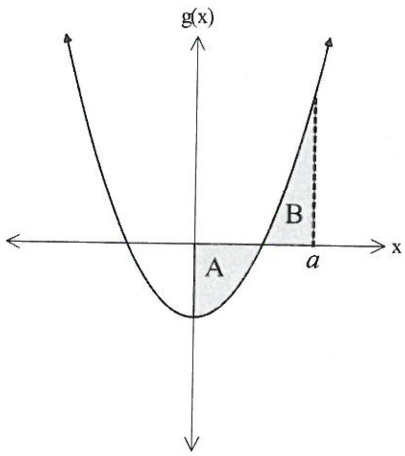
$W(t) = \frac{-(x^3 - 30x^2 - 1800x)}{225} + c$

$W(0) = 200 \quad \therefore c = 200$

$W(24) = \underline{407.36}$

Question 5 [3 marks]

A part of the function $g(x) = x^2 - 4$ is shown. The area of the region marked A is the same as the area of the region marked B. Evaluate the exact value of *a*



$\int_0^2 x^2 - 4 \, dx = -\frac{16}{3}$

$\int_2^a x^2 - 4 \, dx = \frac{16}{3}$

$\Rightarrow \left[\frac{x^3}{3} - 4x\right]_2^a = \frac{16}{3}$

$\frac{a^3}{3} - 4a - \left(\frac{8}{3} - 8\right) = \frac{16}{3}$

$a = 2\sqrt{3}$

Question 6 [2, 4, 3 marks]

An object moves along the *x*-axis with acceleration $a = (3t - 2)\, m/s^2$. Initially it is at the origin and moving with speed $7.5\ m/s$ in a *negative* direction.

(a) Find an expression for velocity in terms of *t*.

$V = \frac{3t^2}{2} - 2t + c$ ✓ at $t=0$ $V=-7.5$

$\therefore V = \frac{3t^2}{2} - 2t - \frac{15}{2}$ ✓

(b) When and where does the object change direction?

$V = 0$ ✓ at $t = 3$ ✓

$S = \frac{t^3}{2} - t^2 - \frac{15t}{2} + c$ at $t=0$ $S=0$

So $S = \frac{t^3}{2} - t^2 - \frac{15t}{2}$ ✓

at $t=3$ $S=-18$ ✓

at $t=3$, the object is 18 units left of the origin.

✓ - puts $V=0$
✓ - time when $v=0$
✓ - antidiff V
✓ - finds where

(c) How far does the object travel in the first 5 seconds?

$\int_0^5 \left| \frac{3t^2}{2} - 2t - \frac{15}{2} \right| dt$

$= 36$ units

✓ - $\int_0^5$
✓ - $| \ |$ or breaking up
✓ - answer

Question 7 [1, 2, 3 marks]

Certain medical tests require the patient to be injected with a solution containing 0.5 micrograms (μg) of the radioactive substance Technetium-99. This material decays according to the rule: $T = T_0 e^{-0.1155t}$ where *t* is the time in hours from injection.

a) What is the value of T_0?

0.5 ✓

b) What is the half-life of Technetium-99?

$0.5 = e^{-0.1155t}$ ✓

$t \approx 6$

half-life is 6 hours ✓

c) How long is the amount of Technetium-99 left in the patient's system less than 1% of the initial amount? Give your answer to the nearest hour.

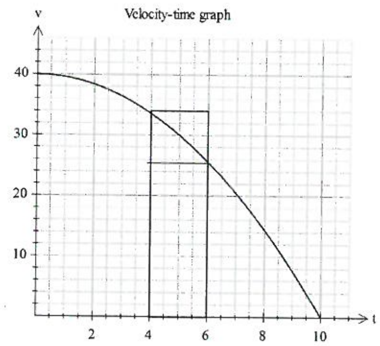
$e^{-0.1155t} < 0.01$ ✓

$t = 40$ hours ✓

Question 8 [5, 1 marks]

A train is travelling at 40metrers per second when the brakes are applied. The velocity of the train is given by the equation $v = 40 - 0.4x^2$, where t represents the time in seconds after the brakes are applied.

The area under a velocity-time graph gives the total distance travelled for a particular time period.



a) Complete the tables below and estimate the distance travelled by the train during the first six seconds by calculating the mean of the areas of the circumscribed and inscribed rectangles. (The rectangles for the 4–6 seconds interval are shown on the grid above.)

Time (*t*)	0	2	4	6
Velocity (*V*)	40	38.4	33.6	25.6

✓

Rectangle	0-2	2-4	4-6	Total
Circumscribed area	80 ✓	76.8	67.2	224
Inscribed area	76.8 ✓	67.2	51.2	195.2 ✓

Estimated total distance travelled = 209.6 ✓

b) Describe how you could better estimate the distance travelled by the train during the first six seconds than by the method used in part (a).

Narrower rectangles

or $\int_0^6 40 - 0.4x^2 \, dx$ ✓